

## Review

## Digital twin modeling

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## ABSTRACT

The digital twin is an emerging and vital technology for digital transformation and intelligent upgrade. Driven by data and model, the digital twin can perform monitoring, simulation, prediction, optimization, and so on. Specifically, the digital twin modeling is the core for accurate portrayal of the physical entity, which enables the digital twin to deliver the functional services and satisfy the application requirements. Therefore, this paper provides systematic research of current studies on the digital twin modeling. Since the digital twin model is a faithful reflection of the digital twin modeling performance, a comprehensive and insightful analysis of digital twin models is given first from the perspective of the application field, hierarchy, discipline, dimension, universality, and functionality. Based on the analysis of digital twin models, current studies on the digital twin modeling are classified and analyzed according to the six modeling aspects within the digital twin modeling theoretical system proposed in our previous work. Meanwhile, enabling technologies and tools for the digital twin modeling are investigated and summarized. Finally, observations and future research recommendations are presented.

## 1. Introduction

The increasing production intricacy in a more demanding market is gathering momentum for the integration of the physical and digital world. At the same time, human's escalating practical needs for industrial products are challenging the digital model's capability to interact with the physical object. The digital twin was conceived in this context and has sparked a far-reaching industry revolution [1,2]. In 2003, the concept of the digital twin was originally proposed by Michael Grieves [3] in his product lifecycle management course at the University of Michigan. In the beginning, digital twins were employed primarily in the military and aerospace. Currently, the digital twin is in a period of rapid development [4]. As seen in Fig. 1, more than 1000 papers concerning the digital twin have been published in each of the three consecutive years since 2019. By the end of 2021, 2934 papers were published in the year. Besides the surging number of papers year by year, the digital twin was listed as top 10 strategic technology trends by Gartner company for three consecutive years, ranking No. 5 in 2017 [5], and No. 4 in 2018 [6], 2019 [7]. In 2020, Gartner also listed the digital twin as an emerging technology for the next 5–10 years. At that time, representations of the digital twin for the real and virtual worlds will be ubiquitous [8].

The digital twin has progressed from theoretical research to pragmatic implementation, whereas the model is a paramount constituent of the digital twin and a prerequisite for successful digital twin applications. In virtual space, based on the attributes of the physical entity, the digital twin model can be expressed in four model dimensions: geometry, physics, behavior, and rule [9]. The geometric model describes the geometric shape and assembly relationships of the physical entity. The physical model reflects the physical properties, characteristics and constraints of the physical entity. The behavioral model represents the dynamic behavior of the physical entity in response to the internal and external mechanisms. The rule model incorporates historical data and can exploit tacit knowledge, making the digital twin model smarter. By incorporating multidisciplinary knowledge, the multidimensional digital twin model can perform functions like prognostication, optimization, and control in the digital world.

However, the effective digital twin model needs to be built from multiple digital twin modeling aspects. In our previous work [10], the digital twin modeling theoretical system was proposed, which deconstructed and investigated the digital twin modeling from the six aspects of model construction, model assembly, model fusion, model verification, model modification, and model management. Model construction integrates the knowledge in related disciplines to construct the

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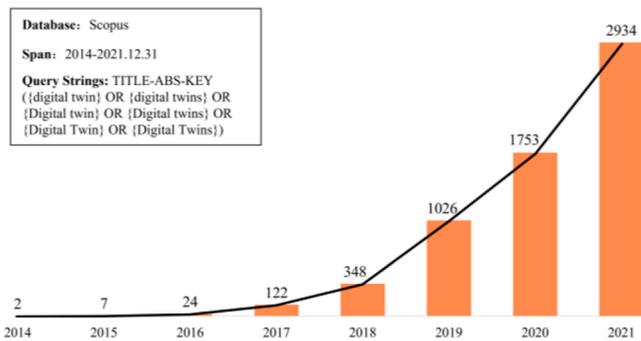


Fig. 1. Number of papers published per year during the digital twin growth stage [4].

elementary digital twin model, like unit level digital twin model in a specific application field. More complex digital twin model can be obtained through model assembly of elementary digital twin models in the spatial dimension. In case assembled model in spatial dimension cannot portray the internal mechanisms of the object or the production process comprehensively and incisively, model fusion in multiple fields, disciplines, and model dimensions is required after model assembly. Model verification for the constructed and assembled or fused digital twin model is also a necessity. If the digital twin model fails to address the physical production requirements, further model modification is essential to secure the accuracy of the digital twin model. Crucial as well, model management provides services, including the management of digital twin models in the model library and the processing and utilization of data and knowledge generated in the digital twin modeling. Hence, as the basis for the digital twin model-driven approach and imperative requirement for digital twin functionality implementation, the digital twin modeling drives the practical deployment of the digital twin technology in the relevant domains and industries.

In this paper, a systematic review of current research on the digital twin modeling and its result, i.e., the digital twin model, will be presented mainly in terms of the four model dimensions and six modeling aspects mentioned above.

### 1.1. Related review works

The digital twin has been an ongoing area of concern in various industries recently. Correspondingly, a high volume of publications on the digital twin modeling have been published, of which several reviews.

Barricelli et al. [11] reviewed current studies on the definitions of the digital twin and analyzed the difference between them. Lim et al. [12] summarized the concepts of the digital twin and the related techniques within the full life cycle. In the design phase of the full life cycle, Leng et al. [13] overviewed the digital twin frameworks. Errandonea et al. [14] conducted further review studies in the digital twin maintenance phase of the full life cycle.

These studies reviewed the digital twin definitions, concepts, and frameworks from various viewpoints, such as the full life cycle. However, as key elements for the digital twin applications, the modeling and model require more focused research. Therefore, a systematic review of the digital twin modeling and model from multiple perspectives is needed.

Melesse et al. [15] performed a systematic literature review to provide an assessment of the utility of the digital twin model in industrial operations and highlight the challenges in its implementation. In the review work of Wright et al. [16], they underlined the essence of the differences between models and digital twins, overviewed some crucial advantages of employing digital twin models, and recommended orientations for further research to explore its maximum potential. Lechler et al. [17] overviewed and explained the relationship between the functionality of the digital twin model and its application fields.

Meanwhile, a new digital twin model in the manufacturing context was introduced. Berri et al. [18] provided a review of digital twin models regarding the fluid-dynamic behavior in electro-hydraulic servo valves. The comparison and evaluation of the precision of digital twin models were illustrated as well.

These publications conducted detailed surveys of the functions of the digital twin model for different objects or scenarios in manufacturing. However, current review works focus on the functional representation of the digital twin model at the surface layer, while overlooking the analysis of the model attributes and features at the deep layer, which shapes the digital twin model. Meanwhile, digital twin models have been evolving and applied in various fields, but the current review works focus only on the manufacturing domain. It is absent a thorough and insightful summary of the similarities, differences, and connections between the multiple attributes and functionalities of digital twin models in various application fields.

Rasheed et al. [19] reviewed the current methods and techniques used in the model construction aspect of the digital twin model. The existing challenges associated with the digital twin model construction were also elucidated, along with the authors' informative proposals and insights. Bordeleau et al. [20] reviewed diverse model-driven engineering technologies used in model construction that potentially contribute to solving the digital twin modeling challenges mentioned. Ríos et al. [21] offered a digital twin modeling review about measurement uncertainty within data transfer standards and how it relates to testing data when constructing the model. Andronas et al. [22] summarized and addressed the limitations of existing techniques for the digital twin modeling of the flexible material.

For system-level digital twin models such as production equipment or the shop floor, their modeling is complex system engineering which is infeasible to construct the entire model in one go. Therefore, based on proper model management, it is necessary to assemble and fuse unit-level digital twin models and verify and modify the modeling validity. These papers reviewed the digital twin modeling perceptively from different viewpoints. However, current studies stagnated in model construction and neglected other aspects of the digital twin modeling including model assembly, model fusion, model verification, model modification and management. Also, related reviews lack a summary and analysis of the specific aspects of the digital twin modeling and the enabling technologies and tools in each aspect. Moreover, the existing research on technologies and tools for the digital twin modeling is discrete and divorced, where no universal and coherent analysis, generalization, and elaboration for both is in progress.

### 1.2. Purpose of this paper

The digital twin model is the foundation and container for the digital twin functionality, while the digital twin modeling is the groundwork for the digital twin model-driven methodology. Therefore, via an exhaustive and incisive overview, this paper aims to:

- 1) Provide a comprehensive summary and analysis of the currently available digital twin models in terms of the application field, hierarchy, discipline, dimension, universality, and functionality.
- 2) Classify and analyze the existing research on the digital twin modeling according to the six modeling aspects within the digital twin modeling theoretical system.
- 3) Investigate the enabling technologies and tools accessible in the six digital twin modeling aspects.
- 4) Propose future research orientations and prospective approaches to resolving existing problems and emerging challenges in the digital twin modeling.

The rest of this article is organized as follows. Section 2 introduces the literature classification criteria and review methodology. Section 3 analyzes the digital twin model from multiple perspectives. Section 4

researches the digital twin modeling from multiple aspects. Section 5 summarizes enabling technologies and tools used in different aspects of the digital twin modeling. Several observations and future research recommendations are given in Section 6. At last, the conclusions and future work are provided in Section 7.

## 2. Literature review methodology

### 2.1. Classification criteria

The fundamental literature classification criterion in this study is based on the application field of the digital twin model. digital twin models in various application fields are deconstructed, classified, and summarized from the perspective of the hierarchy, discipline, dimension, universality, and functionality. As a way to realize the digital twin model in different fields, the digital twin modeling in the publications reviewed is deconstructed and categorized into corresponding modeling aspects within the digital twin modeling theoretical system. Further dissections are taken in each aspect as well. Based on the paper classification for the digital twin modeling, the enabling technologies and tools used in each modeling aspect are researched.

### 2.2. Literature review methodology

Our review is well-grounded in the scientific publications including journal articles, conference papers, book chapters, reviews, and editorials, of which the reviews with reference significance have been outlined in the introduction. The review is undertaken under four procedures: (1) Retrieve the Scopus database by advanced document search with specific query strings to obtain the original retrieved document results, totaling 331. (2) Eliminate extraneous publications by reviewing detailed content. The pertinence of each retrieved publication to the research themes (e.g., attributes of the digital twin model) is evaluated based on the abstract, introduction, and conclusion. For instance, despite the keywords "digital twin" and "model" included in the publication title, its specific content emphasized the similarities and dissimilarities between the digital twin technology and model-based systems engineering rather than the digital twin modeling. Similarly, the authors removed such irrelevant publications from further review. In total, 296 academic publications are found after filtering out the extraneous ones and reviews. (3) Review the entire content of all relevant publications filtered in step 2 and contextualize them under the criteria presented in subsection 2.1. (4) Undertake the insightful analysis depending on the application fields, attributes, and functionalities of the digital twin model, different modeling aspects within the digital twin modeling theoretical system, and related technologies and tools in the literature selected. The relevant data statistics and analytics will be illustrated in later sections. Table 1 shows the database, query strings, original retrieved document results, papers filtered in step 2, and the time frame of the literature review.

**Table 1**  
Retrieval index of the literature review.

Retrieval index	Detail content
Database	Scopus
Query strings	TITLE ({digital twin} OR {digital twins} OR {Digital twin} OR {Digital twins} OR {Digital Twin} OR {Digital Twins}) AND TITLE ({model} OR {models} OR {modeling} OR {modelling})OR {Model} OR {Models} OR {Modeling} OR {Modelling})
Document results	331
Papers filtered	296
Time frame	2003.1–2021.6.28

## 3. Multi-perspective analysis of the digital twin model

Among these 296 academic publications, the 12 publications mainly focus on the macroscopic framework, modeling procedures and methodologies, the technologies, and the tools used in modeling, which will be covered in later sections. In the remaining 284 publications, digital twin models derive from architectures or paradigms and application examples in case studies. As shown in Fig. 2, these publications are classified by the application fields of their digital twin models. Because one reviewed publication may concern more than one application field, the publication will be counted in each statistic data of its application fields which causes the total percentage of the fan diagram to be greater than 100 %. In the meantime, the 284 publications related to diverse application fields will serve as the principal basis for the statistics and analytics in the subsequent sections.

### 3.1. Application field analysis of digital twin models

Digital twin models have been successfully implemented in a wide spectrum of manufacturing, such as from full-scale factories to precision parts, from macro manufacturing systems to specific product production processes. From Fig. 2, nearly half of the reviewed publications are related to manufacturing. The application of digital twin models in manufacturing is not only wide-ranging but also detailed and subtle. A variety of workpieces or parts make up a piece of integral equipment with ad hoc utilities. Dissimilar equipment is used in production lines or shop floors for diverse production purposes. For robotics alone, the digital twin models can be categorized into several sub-applications (industrial robot [23–26], mobile robot [27], line-following robot [28], robot arm [29]) which are ready for specific production scenarios. Except for manufacturing, digital twin models are working in fields like energy, aerospace, engineering construction, city, healthcare, agriculture, and so on. Second only to manufacturing, applications of digital twin models in the energy field (35 papers) can be further refined into energy exploitation and energy source equipment. The third major application field is aerospace (20 papers) including flight vehicles and parts, equipment, or systems associated with them. In addition, digital twin models have specific uses in other areas such as earthquake monitoring [30], heritage conservation [31,32], and naval vessel repair [33].

Although the digital twin model has been effectively practiced in many fields, in some fields, it can currently only be applied in a narrow range of scenarios. For example, the current digital twin technology applied in the apparel field is only limited to the reconstruction of historical garments [34]. In future research, virtual design and try-on of the apparel can be achieved by constructing digital twin models for apparel in various styles to reduce the cost of physical design and enhance the try-on experience. In the chemical industry, the digital twin model applications are dedicated to the process and device level, i.e., phosphate slurry piping process [35], rectification installation [36], and combustion furnace [37]. Therefore, the construction of digital twin models for specific chemical materials or chemical molecules and atoms at a more microscopic level is a seminal breakthrough for future digital twin technology research. Besides narrow application scenarios, digital twin models in some fields also suffer from uneven hierarchical distribution, insufficient multidisciplinary integration, one-sided study on model dimensions, and a gap between the functions provided and the actual needs, which will be elaborated on in later sections.

### 3.2. Hierarchical analysis of digital twin models

Physical entities are hierarchical and the digital twin model is a virtual mapping of the physical entity. Accordingly, based on the concept of the digital twin shop floor hierarchy proposed by Tao et al. [9], digital twin models surveyed in this review can be generally classified into unit level, system level, and system of systems (SoS) level by

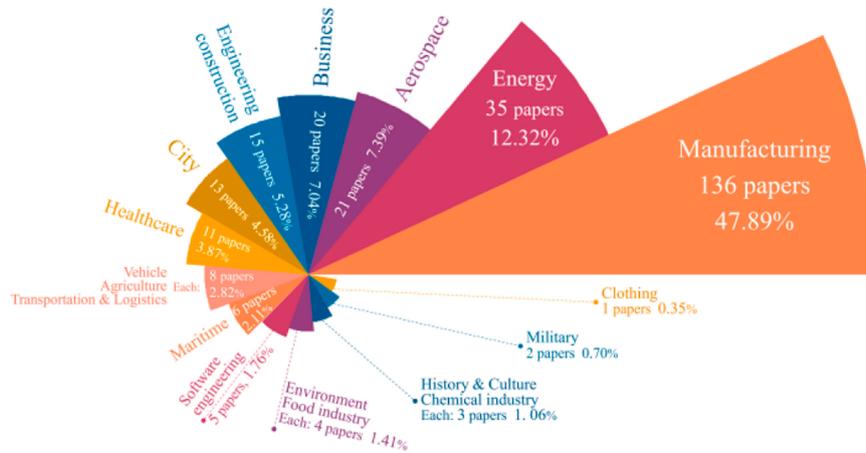


Fig. 2. Application field distribution of digital twin models.

their structure and functionality.

For a penetrating perception of the hierarchical structure of the digital twin models in manufacturing, the hierarchy can be divided into production-oriented and product-oriented ones.

(1) Production-oriented hierarchy of the digital twin model: In production-oriented hierarchy, equipment, production lines, and shop floors constitute unit level, system level, and SoS level, respectively. Unit-level digital twin model for individual equipment enables functions like monitoring, fault prediction, maintenance, etc. Production lines and shop floors can be modeled to represent the interaction and coupling relationships between subsystems in favor of analyzing and predicting the evolution of the whole system. Among the 284 publications, 51 reflect the hierarchical theory, visualized in Fig. 3. Current research requires continued advancement not only at the unit level but also at the system and SoS level. Based on an evenly distributed hierarchy, each level can be incorporated level by level to realistically portray the physical shop floor. Besides uneven hierarchical distribution, the existing connections between levels mainly stay at the geometric dimension, with weak coupling and poor linkage in physical or logical dimensions.

(2) Product-oriented hierarchy of the digital twin model: In addition to being a production unit in the production-oriented hierarchy, production equipment has its hierarchical structure as a product. There are several subsystems in the machine tools. For example, the digital twin mechanical machine tool contained the main body, spindle system, and feed system. Objects with specific functions, like drivers, motors, etc., were contained in the feed system [25]. Here, the mechanical machine tool is at the SoS level with its subsystems at the system level. Functional execution units, such as motors, are at the unit level. Few studies now investigate a product-level digital twin model from the perspective of a product-oriented hierarchy. The integrity of the digital twin model will be compromised without an accurate portrayal of its sub-models and the connections between them at each level of the product. This adversely affects the functionalities of the digital twin model and consequently the

production-oriented hierarchy where it operates.

The concept of the digital twin shop floor hierarchy is not limited to manufacturing, we extend the concept to fields like aerospace, city, and healthcare. Table 2 shows their hierarchical distribution, and the respective analysis is performed below. In the field of aerospace, the flight vehicle as a whole, we define as the SoS. The subsystem constituting the flight vehicle, such as the propulsion system, is defined as the system. The component that performs the functions of the subsystem is defined as the unit. The current studies are not comprehensive for the hierarchical structures of digital twin models of the flight vehicle. In particular, system-level models require to be pioneered and well investigated. In the field of city, the individual urban construction is considered as the unit. The building complex, such as industrial estate and residential areas, is at the system level. Finally, the whole city is at the SoS level. The city-oriented digital twin models are majorly distributed at the unit level and SoS level. However, there is a complete vacuum in the study at the system level. Therefore, system-level digital twin models, such as commercial, industrial and residential areas, are new directions for future research.

Besides the varying degree of success achieved in the engineering area, the digital twin model in healthcare is receiving increasing attention [68]. With the “best-fit” feature of the digital twin technology, digital twin models within healthcare setting will be a break from established care paradigms [69]. It was reported that a quarter of healthcare executives experimented with the digital twin in 2021 and 66 % of them indicated their investment in the digital twin will increase over the next three years [70]. With significant application value and broad application prospects, the digital twin model in healthcare is

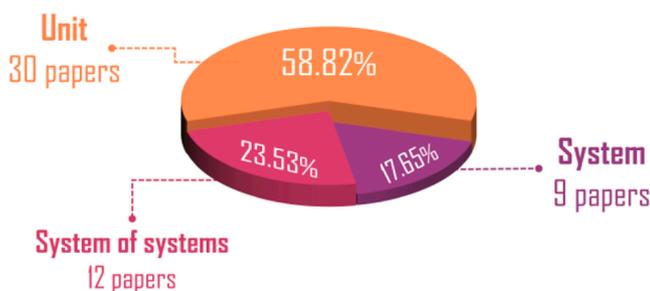


Fig. 3. Hierarchical distribution in the digital twin shop floor.

Table 2  
Hierarchy of the digital twin model in other fields.

Field	Unit	System	SoS
Aerospace	Aircraft tire [38]	N/A	UAV [46–48]
	Air rudder [39,40]		Spacecraft [49]
	Aero-engine bearing [41]		Rocket [50]
	Aircraft cabin [42]		
	Hydraulic valve [43]		
	Air bearing [44]		
	Turbofan [45]		
City	Utility pole [51]	Campus [60]	City [61–64]
	Bridge [52–54]		
	Historical architecture [31–34]		
Healthcare	Building [55–59]		
	Coronary heart vessels [65]	Cardiovascular system [67]	N/A
	Heart [66]		

analyzed according to its hierarchical structure, which can be further subdivided into human-oriented and medical resources-oriented hierarchies.

(1) Human-oriented hierarchy of the digital twin model: We define the overall human body as the SoS from a biologically and medically-based perspective. Then the eight systems, for example, the cardiovascular system, making up the human body, are at the system level. Lastly, the human organ is defined as the unit. Although digital twins applied in healthcare is a popular research trend nowadays, there are rarely successful implementations, with only coronary heart vessels [65] and heart [66] at the unit level, and cardiovascular system [67] at the system level. Even more unfortunate is the lack of exhaustive and precise digital twin models for the entire human body. Since the digital twin technology perfectly corresponds to the brand-new health philosophy today, also known as preventative medicine, the study of digital twin models in the medical field will be a compelling facilitator for future medical concepts.

(2) Medical resources-oriented hierarchy of the digital twin model: For a magnetic resonance imaging machine, the entire equipment is SoS and its subsystems such as the radio frequency (RF) system are at the system level. Correspondingly, the functional units constituting the RF system such as RF coil, RF generator, etc., are at the unit level. The digital twin model hierarchies of medical resources enable real-time monitoring and interconnection of devices and even parts on each level of the device. As a result, it is expected to transform the device

maintenance from responsive to anticipatory, while cutting hospital operation and management costs. No papers have yet addressed the medical resources-oriented hierarchy of the digital twin model—as a promising research direction.

### 3.3. Disciplinary analysis of digital twin models

Among the 284 publications, 137 are selected where relevant disciplines were specifically considered in digital twin models. Based on relevant disciplinary backgrounds or oriented to relevant disciplinary problems, these publications utilized relevant disciplinary methodologies, technologies, or tools in the digital twin modeling, or their digital twin models incorporated relevant disciplinary knowledge or solved relevant disciplinary problems in practical applications. Therefore, some publications included more than one discipline. These publications will be counted in each statistic data of their disciplines which causes the total percentage of the fan diagram to be greater than 100%. Collectively, those selected publications encompass 43 distinct disciplines displayed in Fig. 4. It can be noted that digital twin models embrace a wide range of disciplines and become interdisciplinary studies. For example, electro-mechanical-hydraulic integration has always been one of the crucial development directions of smart manufacturing. The employment of digital twin models further enhances the electro-mechanical-hydraulic integration within large equipment like precision machine tools [71–80], making them more automated and

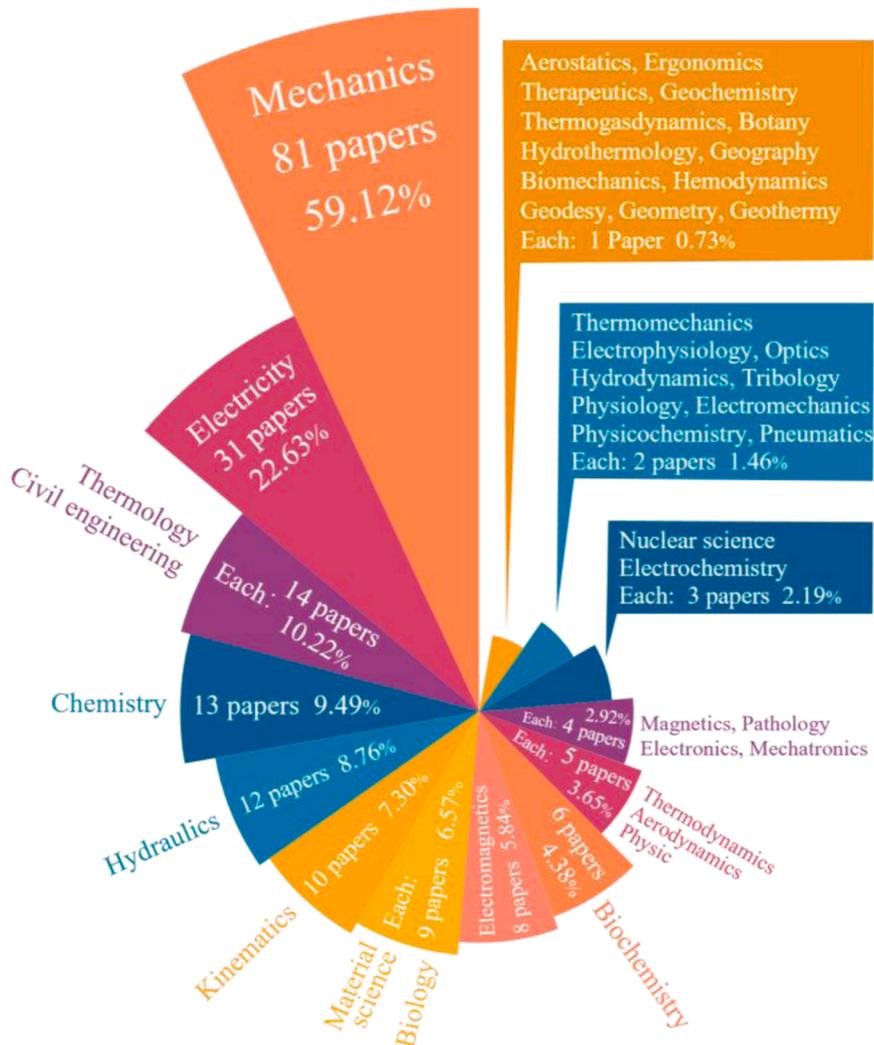


Fig. 4. Disciplinary distribution of digital twin models.

intelligent. Machine learning arising from interdiscipline can predict protein 3D structures in the medical field [81], and their seamless integration within the digital twin model allows an extension in prediction performance.

In general, the digital twin is of the enormous potential to innovate traditional disciplines, empower emerging ones, and boost the research in highly sophisticated ones. However, existing research remains concentrated on manufacturing-related disciplines, such as mechanics and electricity. The extensions to emerging disciplines like data science and interdisciplinary ones like molecular biology will drive the digital twin technology into a new era.

### 3.4. Dimensional analysis of digital twin models

Tao et al. [9] proposed five-dimensional digital twin model, including physical entity, virtual model, connection, data, and service. The virtual model is composed of sub-models in four dimensions: geometric model, physical model, behavioral model, and rule model. These sub-models are equipped with their own characteristics and functions. By assembling or fusing the above sub-models, the virtual mapping of the digital twin model for the physical entity can be achieved. Among the 284 publications, 126 in 15 application fields, ranging from manufacturing to vehicle and environment to business, are further screened. In these publications, their digital twin models partially or completely reflected the four model dimensions.

63 of the 126 publications with digital twin models correlate to manufacturing. Fig. 5 presents that the digital twin model research towards the four model dimensions was studied in a relatively balanced manner. It is worth noting that there are 29 publications whose digital twin models incorporated the full four dimensions. As an example from [96], the virtual mapping at the geometric dimension for the physical production line was achieved by the 3D model assembly. Static structural modeling created physical models where the physical attributes and structural features of the equipment were reflected. The production behavior of the equipment was described by kinematic analysis. By the model fusion of the equipment behavioral models, the production line rule model was obtained to achieve remote control in virtual space. However, there are still some application examples where the digital twin model is not fully functional because the four model dimensions are not completely represented. In efforts to facilitate digital transformation and upgrade in the manufacturing industry, the establishment of a complete functional digital twin model is a prime priority. Creating the virtual model based on geometric, physical, behavioral, and rule model dimensions is the infrastructure for the materialization of the complete functional digital twin model.

Besides manufacturing, digital twin models with full four dimensions

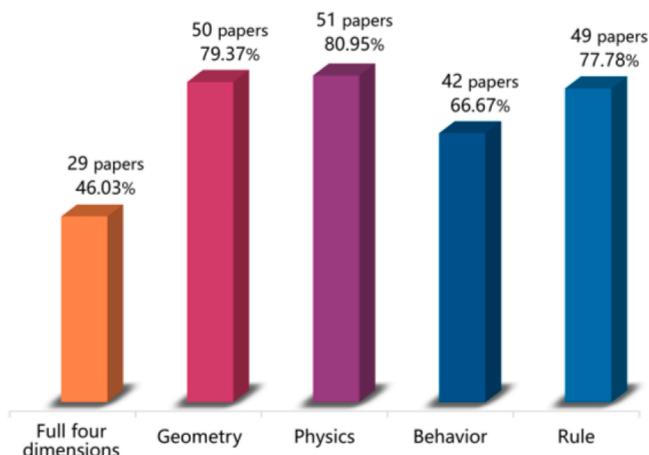


Fig. 5. Dimensional distribution of digital twin models in manufacturing.

can be found in other fields, such as engineering construction, healthcare, city, and vehicle. As an example from healthcare [68], The geometric model was built by the mesh according to the peculiarities of the coronary heart vessel. The grid structure enabled the physical model to reflect the properties of the vessel, like the presence or absence of vortices, the degree of dissipation, and the blood distribution. The behavioral model incorporated a system of differential equations to describe the flow process in the vessel. The rule model can predict the heart diseases progression based on the stress-strain properties and flow process. But in general, the four model dimensions are not comprehensively and adequately conducted in these fields. Some dimensions received disproportionate consideration, but others garnered limited attention. Therefore, future research needs to be explored more intensively and systematically based on methodical approaches towards the four model dimensions, thus leading to a more multipurpose and accurate digital twin modeling.

### 3.5. Universality analysis of digital twin models

Based on the applicable scope of the digital twin model in the publications reviewed, the universality of the digital twin model is studied in this section. Digital twin models are further selected and summarized into two categories: 1) specific one peculiar to an object/scenario; 2) generic one available for application fields.

(1) Specific digital twin model: Oriented by the actual needs or problems from specific application objects and scenarios, the publications, as shown in Fig. 6, adopted a method of procedure with relevant technologies or tools in the digital twin modeling, which make specific digital twin models meet the actual needs or solve the actual problems. Individuality, effectiveness, and applicability are the distinguishing features of specific digital twin model, which were validated by the application requirements from certain objects or scenarios. Nevertheless, there are challenges for mathematical modeling to maintain precise consistency between the digital twin model and sophisticated physical system. To some extent, it is arduous for a specific digital twin model to fulfill all the demands from certain application scenarios. Meanwhile, specific digital twin models may undermine the accurate mapping of physical assets and desired efficacy. However, specific digital twin models for different application scenarios and even distinct domains can be assembled or fused to address such challenges based on the corresponding constraints and coupling relationships. The details regarding assembly and fusion will be given in Section 4.

(2) Generic digital twin model: The generic digital twin model is expressed as the framework or architecture at the macro level whose concepts, paradigms, or know-what can guide the development and use of digital twin models in specific applications. 55 out of the 284 publications incorporated the generic digital twin models, and the 8 application fields they covered are shown in Fig. 7. Besides the 102 for the specific one and 55 for the generic one, there are 127 out of the 284 publications where the definitions, concepts, and frameworks of the generic digital twin model were referenced and extended. For example, the five-dimensional digital twin model [9] is available for all application fields. Among these 127 papers, researchers have used it in the aircraft assembly in aerospace [82], the pre-warning system in vehicle

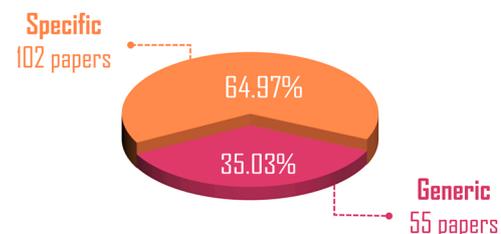


Fig. 6. Distribution of specific and generic digital twin models.

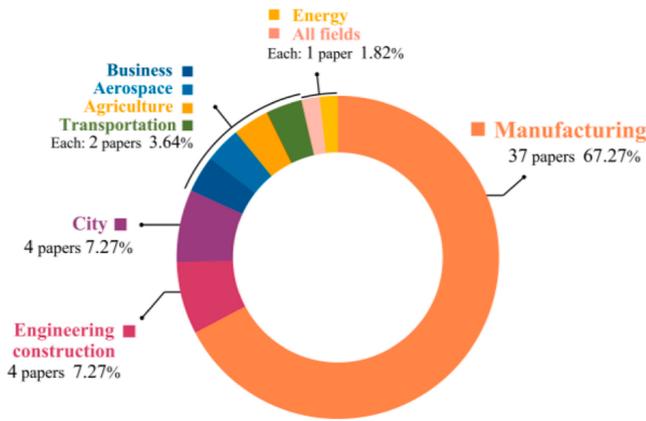


Fig. 7. Application field distribution of generic digital twin models.

[83], the industrial product in manufacturing [84], the transport services in transportation & logistics [85], and the structural design in engineering construction [86]. Notwithstanding the significant advantages of the generic digital twin model over a specific one, such as universality, scalability, and versatility, the standard modeling approaches are prerequisites for successfully unleashing its maximum potential.

### 3.6. Functionality analysis of digital twin models

Beyond the idea or concept stage of what digital twin models could do, 102 of the 284 publications are selected which developed specific functions to solve problems, meet needs, or reach performance in their case studies. From Fig. 8, it is apparent that digital twin models currently offer an extensive functional spectrum with 18 functions. Meanwhile, the digital twin models in some publications are multifunctional. These publications will be counted in each statistic data of their functions which causes the total percentage of the fan diagram to be greater than 100%. The primary functions are visualization, prediction,

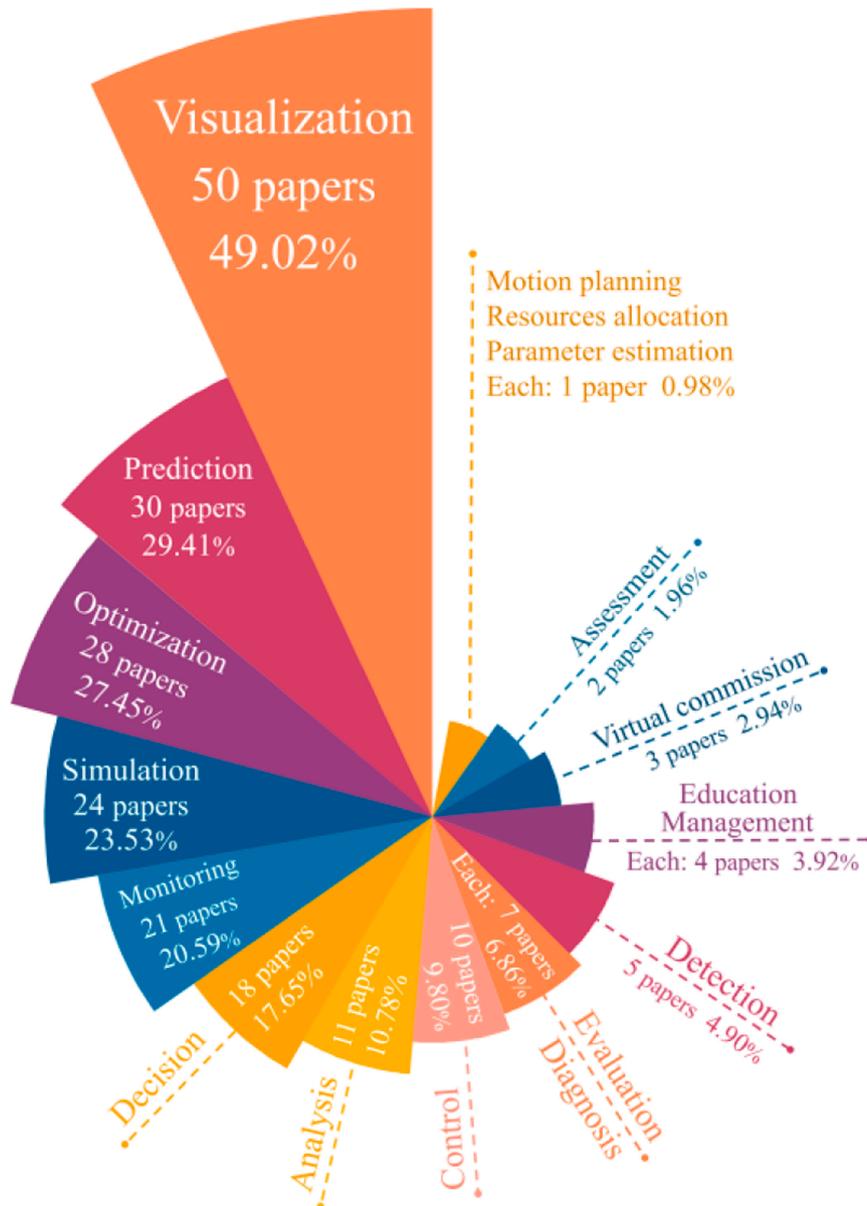


Fig. 8. Functional distribution of digital twin models.

optimization, simulation, and monitoring. Not restricted to the conventional functions, the digital twin model has been expanded to education [87–90], management [91–93], and so forth. Therefore, for the practical and specific requirements of digital twin models in various application fields, it is imperative for the existing conventional functions to be consolidated, intensified, and refined for efficacy maximization. Simultaneously, further exploration of the new functions to provide more content is also required.

More precisely, some more targeted functions are determined by the own attributes of sub-models in different dimensions. The collaboration out of the fused sub-models can better respond to the demands from specific application scenarios. So, the functionality should be analyzed from the unique properties of sub-models in different dimensions and the collaboration between them. After the appropriate iterations of model verification and modification, more effective and efficient functions can be derived.

#### 4. Multi-aspect analysis of the digital twin modeling

The digital twin modeling is digital modeling in virtual space for the properties, methods, behaviors, and other characteristics of the physical asset. In the following section, the digital twin modeling will be illustrated from the six modeling aspects within the digital twin modeling theoretical system with several application cases.

##### 4.1. Model construction

Of the 296 publications filtered in step 2 of subsection 2.2, 90 are selected that highlight the methodologies, technologies, and tools available in model construction. Incorporating knowledge from corresponding disciplines in the particular field, the digital twin model can be accurately constructed from the four model dimensions. From Fig. 9, model construction primarily concentrated on the physical dimension, while the other dimensions received less attention. For advancing future model construction research in a well-balanced manner and straight to the point, this section will analyze and conclude model construction in terms of four model dimensions.

##### 4.1.1. Geometric model construction

Geometric model construction delineates the shape, size, internal structure, spatial position and attitude, and assembly interfaces of physical entities. For geometric model construction, the model fidelity and simplification are worthy of attention.

The geometric model is not only for shaping, but its structural integrity and data accuracy also underpin the motion analysis,

optimized design, virtual interaction, and the like. Therefore, geometric model fidelity is crucial for geometric model construction. For example, Zhang et al. [94] predicted the tool wear state by premium quality visual display and analysis in virtual space.

For fast geometric model transfer, loading, and browsing, an appropriate model simplification method is expected. Geometric model simplification can use fewer data to achieve high fidelity representation of physical entities with small transfer files. It also provides high compatibility across different platforms. For example, Li and Nan [95] proposed a generic model simplification method that retains meaningful mesh structure details while redundant facets are reduced, enabling efficient model edit and analysis for users.

##### 4.1.2. Physical model construction

Physical model construction provides the basis for services such as quality control and the analysis and prediction of the physical property. In detail, physical model construction can classify into static and dynamic ones.

Static physical model construction includes the quantitative modeling of physical property, state, and behavior, which is determined by physical entities solely and independent of varying physical analysis methods. For today's increasingly complex engineering applications, multi-physics coupling analysis is required in static physical model construction. By a multi-physics field coupling modeling approach, Ruan et al. [96] constructed a fluid-thermal-electric multi-physics field coupling model to unveil the reinforcement of thermoelectric properties by nanofluids.

For a dynamically varying physical system, such as thermal conduction inside a mechanical part, a finite number of nodes need to be created and calculated on the Spatio-temporal solution domain to obtain the physical state distribution of the whole system. Based on this methodology, Wei et al. [76] built a finite element model for real-time wear simulation to enable dynamic assessment of performance attenuation of CNC machine tools by digital twin models.

##### 4.1.3. Behavioral model construction

The behavioral model is constructed to represent the sequential, concurrent, linked, periodic, and random behaviors of physical entities. An accurate behavioral model determines the exactitude of the motion and control of the digital twin model. However, physical entities are exposed to a variety of uncertainties in practical operation, and behavioral models tend to be discrepant as a result.

Abnormal values in the data typically undermine the behavioral model accuracy or consequently deviate the digital twin model from the physical entity. Therefore, a precise analysis of the root of anomalous data and its relevance to other variables is essential when constructing behavioral models. For example, Boulfani et al. [97] constructed a more accurate behavioral model by extracting and analyzing the amount of abnormal temperature variation from the physical generator misbehaviors and removing the abnormal behaviors from the digital twin model of the generator.

Meanwhile, adjusting the algorithm parameters to identify the optimal value of each parameter allows for the improvement of the model accuracy. The adjustment of the parameters can be an iterative process on several well-run models. However, parameter tuning entails a deep insight into the parameters' meaning and their impact on the model. Based on the digital twin model and data, Luo et al. [98] studied a hybrid predictive maintenance algorithm with optimal parameters by algorithm adjustments, which realized the prediction of dynamic behaviors of the cutting tool.

##### 4.1.4. Rule model construction

Rule model construction unveils the implicit knowledge and portrays the evolutionary trends and patterns of physical entities. Based on the whole life cycle of physical entities, there are two principal ways of rule model construction, i.e., mining and analysis of whole life cycle data and

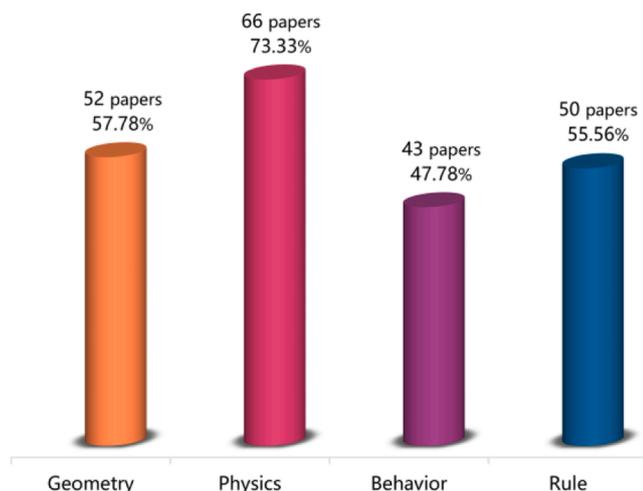


Fig. 9. Dimensional distribution in model construction.

formal representation of experience and knowledge.

The whole life cycle data is the upper limit of the rule model, and the rule model construction is the continuous approximation toward this upper limit. Therefore, for the maximum mapping of the whole life cycle data by the rule model, data pre-processing is required. Scheffel et al. [99] enable the fault detection of the digital twin model based on the pre-processing of raw data from phases of the whole life cycle of the machine.

Through means like data mining, information processing, knowledge measurement, and graphical mapping, complex experience and knowledge are empowered with rule models. Therefore, rule models can reveal dynamic evolution in experience and knowledge domains, which eventually enable digital twin models to understand and apply human intelligence. Lermer and Reich [100] created the digital twin model through the basic knowledge extraction of the dataset from the production process and its mapping to a fuzzy rule set of expertise. Hence, the potential of the digital twin model for machine condition assessment was further exploited by the additive basic knowledge denoted by discovered fuzzy rules.

#### 4.2. Model assembly

Model assembly is a process to realize unit-level digital twin models to the one on higher hierarchies. Model assembly can be realized by adding appropriate spatial constraints of physical assets based on hierarchical relations and assembly sequence between models. For example, Angjeliu et al. [31] identified and geometrically assembled the structural elements of complex architecture regarding the hierarchy from parts to complete model. Unlike traditional CAD model assembly, digital twin model assembly can be dynamically adjusted based on bi-directional data communication in real-time to ensure the high-fidelity mapping of physical assets throughout their life cycle. More specifically, model lightweight before model assembly and interference detection during model assembly are considerations for effective assembly.

##### 4.2.1. Model lightweight for model assembly

The format, information granularity, and modeling accuracy of the models to be assembled are commonly unevenly distributed, causing increased complexity and reduced efficiency in model assembly. Therefore, lightweight digital twin models are expected as long as information integrity, model accuracy, and functionality satisfy the scenario-specific needs. For example, Fang et al. [101] proposed a lightweight method for model assemblies that decrease the geometric errors of simplified meshes and increase the frame rate. The proposed method was validated in several digital twin production scenarios.

##### 4.2.2. Interference detection for model assembly

When conducting model assembly in the virtual space, errors are inevitable in model construction. Consequently, interference problems may occur between digital twin models at the same or different levels, which compromise the accuracy of subsequent model assembly or even failure to assemble. To improve the quality and efficiency of the assembly, interference detection is required during model assembly. Zhang et al. [102] raised the assembly accuracy and efficiency by a method for assembly interference detection based on MRT, where geometric topologies and assembly constraints are no longer in demand.

#### 4.3. Model fusion

Model fusion is a necessity when model assembly in the geometric spatial dimension is insufficient to depict physical objects. For the realistic mapping of the physiological state of the cardiovascular system, Mazumder et al. [70] fused digital twin models of organs and tissue in the cardiovascular system through pulmonary and systemic blood flow regulated by hemodynamic equations. In more detail, model fusion

incorporates the fusion between different model dimensions within a digital twin model, as well as the fusion between digital twin models in different fields.

##### 4.3.1. Fusion between model dimensions within a digital twin model

By clarifying the coupling mode and constructing the coupling relationship beyond the geometric space, the fusion between four model dimensions within a digital twin model can be accomplished. For case in point, Liu et al. [103] created the individual instances of ontology classes in different dimensions and fused them into a semantic net with slot values. In this way, the production system hierarchy for model fusion was constructed.

##### 4.3.2. Fusion between digital twin models in different fields

Besides the fusion within a digital twin model, the fusion of ones with various disciplinary knowledge in different fields is crucial for constructing digital twin models at the system level or SoS level. The digital twin model of the CNC machine, for example, is composed of digital twin models like electrical, hydraulic, and mechanical systems containing various disciplinary knowledge in different fields. The electrical system controls the hydraulic system through a communication interface. The hydraulic system drives the mechanical system through a drive connection. The mechanical system in motion feeds information back into the electrical system. In this manner, the digital twin models of these systems are fused to develop an intelligent closed loop. Similarly, Luo et al. [71] fused the electrical, hydraulic, and mechanical systems into the digital twin descriptive model where fault prediction and diagnosis for the CNC milling machine tool were achieved.

#### 4.4. Model verification

For achieving functional efficacy and meeting requirement consistency, model verification is needed, which ensures that the digital twin model accurately mirrors the actual system. Model verification is to evaluate the consistency between the digital twin model and the physical object output under the same conditions. The unit-level digital twin models are required to be verified first to guarantee a valid foundation for a more complex one. Key points in model verification can be concluded as follows:

##### 4.4.1. Information reconfirmation for model verification

Assess all information relevant to the digital twin model and identifies what additional analysis to improve the model credibility. Therefore, Model verification and modification is an iterative process where the model is continuously refined until satisfactory [10].

##### 4.4.2. Vertical analysis for model verification

Evaluate the model evolution at various historical phases to assess the model suitability for the prospective application. For example, by utilizing the predetermined cost function, Schluse et al. [104] developed iterations with the verification for historical parameters.

##### 4.4.3. Concept and fidelity analysis for model verification

Evaluate the model's algorithms and sub-models in different dimensions to identify the assumptions unbefittingly and sub-model fidelity for the prospective application. In predictive maintenance, sub-model parameters were verified for accurate digital twin modeling according to the deviation exceeding between actual and simulated signals in the predetermined range [23].

##### 4.4.4. Logic tracing for model verification

Evaluate the digital twin model in behavioral and rule dimensions to identify whether individual behaviors and their combinations are desirable. For example, during the tensile tests, the candidates precisely replicating the reduction were identified, otherwise, they will be removed. Then, among identified candidates, the set replicating the

measured force in maximum precision was selected for production simulation in the fracture toughness test [105].

#### 4.5. Model modification

If there is an unacceptable deviation between the digital twin model and the physical object in model verification, model modification is required. Reasonable selection of the parameters for modification is paramount to an effective and efficient model modification. Simultaneously, based on objective functions formulated rationally, the appropriate methodology for modification is required.

##### 4.5.1. Parameter for model modification

To prevent pathological numerical problems in model modification, the number of parameters selected for each modification should be adequate. However, an insufficient selection of parameters will yield less effective modification results. For example, depending on the effect of each modification, a certain amount of model parameters of new porous metal plasticity was selected for updating. Accordingly, the sufficient selection of parameters for modification refined the predictive capability of the digital twin model [105]. Meanwhile, the parameter variables to be modified should select those that are error-prone and physically meaningful in the structure. For example, Zhang et al. [106] used the proxy model to identify the locations of parameters for modification in virtual materials. At the same time, their error probability was evaluated by the generic algorithm to optimize the modification accuracy.

##### 4.5.2. Methodology for model modification

After determining the parameters, a reasonable methodology for modification is demanded. The core of the modification method is to construct an objective function that aligns the digital twin model output as close-to-real results as possible while enabling iterative modifications for the parameters of the digital twin model. In [31], while considering the structural response of the system, dynamic measurements were used for model modification. The modification measured how accurately the model matches the real building, and the iteration of verification and modification improved the accuracy of the model inch by inch. At the same time, Vrabic et al. [27] utilized gradient descent for optimizing the digital twin model output accuracy and control parameters in each cycle until they converged to terminal values.

#### 4.6. Model management

Based on the above digital twin modeling aspects, model management refers to multidimensional and multi-field digital twin model management and model knowledge base management. It is crucial to managing the data generated from the digital twin modeling as well. Therefore, users can derive relevant services from model management. In [46,47], a library for component-based reduced-order models was constructed, where reduced-order models can be rapidly created, adapted, and evaluated. However, the digital twin model attributes and digital twin modeling data are not the entirety of the management, where extensions to the authority, technology, and tool are possible.

##### 4.6.1. Authority management for model management

For security and convenience reasons, some functions and services provided by model management need to be restricted and customized for different users. Therefore, authority management is needed, which includes the authority to browse or utilize digital twin models and their data at each digital twin modeling aspect, the authority to monitor the model status and receive feedback after the digital twin model is deployed to production, and so on.

##### 4.6.2. Technology and tool management for model management

Inevitably, a multitude of technologies and tools are involved in each

digital twin modeling aspect. Rational management is required to guarantee that modeling is satisfied promptly with a system of high-quality, efficient, packaged technologies and tools. The management should include the feasibility, functionality, reliability, efficiency, maintainability, usability, and portability of the modeling technologies and tools. Moreover, it in turn facilitates the management and utilization of the data and empirical knowledge generated during the digital twin modeling.

#### 4.7. Statistics analysis toward the digital twin modeling

The 120 papers in 16 application fields are selected and classified according to the six modeling aspects within the digital twin theoretical modeling system. Meanwhile, nearly half of them (58 papers) is concerned with the manufacturing field. From Fig. 10, model construction has 45 papers related and remains the highest proportion among the six modeling aspects at 77.59%. However, there is still neglect of model management, with only three papers [107–109] on it, a mere 5.17% of the six modeling aspects. It is evident that the current research on modeling for the digital twin model mainly restricts to model construction and involves less on the other modeling aspects, especially the fewest research on model management. For establishing a complete digital twin model to adapt to the corresponding intricate application scenarios, it is advisable to reinforce the research on the five modeling aspects besides model construction.

Except for manufacturing, only in aerospace, all six modeling aspects within the digital twin modeling theoretical system are accessible. However, the corresponding papers are far less frequent. For other fields, all modeling aspects are not thoroughly addressed, with the exception that they all shed light on model construction, e.g., model construction in the field of the city [51,55,60,61,110,111], and model construction in the chemical industry [35,36]. Simultaneously, the papers are attainable whose general methodologies are feasible for one and several modeling aspects within all application fields ([112] for model fusion, [113–117] for model management, and [118] for model construction, verification, modification, and management).

### 5. Enabling technologies and tools in the digital twin modeling

There is still a mountain of challenges for technologies and tools that remain to be resolved for the digital twin modeling. Meanwhile, current research concentrates on frameworks, processes, and know-what at the macro level rather than specific technologies, tools, and know-how at the micro-level [119]. Furthermore, the digital twin modeling is a highly intricate process that entails a time-intensive orientation, fine-tuning, and refinement. This section, therefore, surveys and studies the enabling technologies and tools used in the six digital twin modeling aspects, which serve as technological and tool references for the digital twin modeling and consequently facilitate the digital twin implementation by researchers and practitioners.

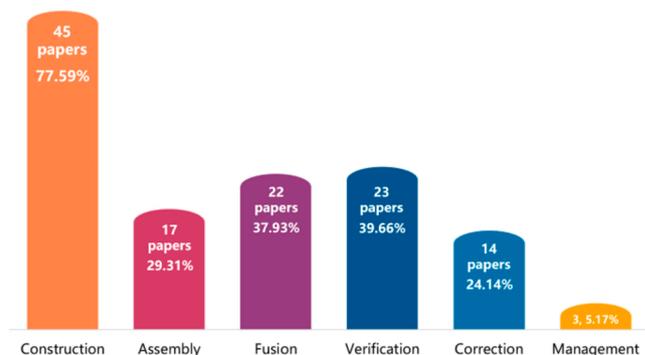


Fig. 10. Distribution of digital twin modeling aspects in manufacturing.

5.1. Enabling technologies and tools in model construction

With disciplinary knowledge in multiple application areas, model construction establishes a model foundation on four dimensions for complex digital twin models. In this part, the technologies and tools implemented in model construction are investigated from the four modeling dimensions.

5.1.1. Enabling technologies and tools for geometric modeling

Geometric modeling technologies portray geometric appearance, which mainly address the geometric and topological information of the physical entity. In the literature reviewed, point cloud [31,32,58,60,93,109,120] and building information modeling (BIM) [31,32,58,93] are the most frequently used geometric modeling technologies. Point cloud is a dense collection of points collected by certain measurement means. The point cloud derived from laser scanning [31,32] and photographic scanning (Photogrammetry [32], Pinhole camera modeling [51]) is called a dense point cloud, where the number of points is larger and denser. Through point cloud, the target surface can be characterized, and finally, 3D physical entities are represented directly and efficiently. BIM incorporates the technological merits of static structural modeling [103] and object-oriented modeling [71,73] to create 3D models with non-graphical features and information. Compared to GIS analysis technique [30] focusing on the macro level, BIM covers the micro-level. By integrating the efficient data acquisition of point cloud and the macro geometric spatial concept of GIS, BIM will fulfill more ambitious digital twin scenarios like digital twin cities. Apart from the above, finite element modeling technology, shown in Fig. 11, was used for geometric modeling of the historical building [31] and machine tool [76].

3D geometric models are commonly constructed by specialized tools such as 3D modeling software. Shown in Fig. 11, Pro/E [103], CATIA [121], SolidWorks [82,87,122,123] are the 3D modeling software dedicating to industrial machinery. Autodesk REVIT [57] and Surfer [30] are the ones focusing on building information modeling and 3D geographic information modeling, respectively. Reverse modeling tools, such as LiDAR scanner [52], Zenfone AR [124], Kinect sensor [29], Laser scanner [93], enable geometric modeling of physical entities as well. Furthermore, importing models constructed by the above software and tools into platforms like Flexsim [91], Abaqus [106], Demo 3D [125], Unity3D [89,126], and Siemens NX [127,128] can perform more

complex geometric information construction and animation production. For web-side geometry modeling, web graphics library (WebGL) [129] combined open graphics library (OpenGL) [108] and the programming language of JavaScript [129] to provide 3D graphics application programming interfaces (APIs). To deliver digital twin services at people's fingertips, 3D rendering of mobile web pages with better cross-platform capabilities are imperative. By simplifying and encapsulating WebGL, Three.js [103,129], with its user-friendliness and low system resource consumption, will boost the 3D modeling of web pages on mobile in the future.

5.1.2. Enabling technologies and tools for physical modeling

Physical modeling technologies are used to delineate the physical features and constraints based on geometric modeling. For physical modeling, the most commonly used technology is finite element modeling [32,76,77,106,130–134], using mathematical approximations to develop models of materials, loads, constraints, and mechanical properties of the physical entity. The core of finite element modeling is discretization. However, discretization operations are positively correlated with the complexity of the physical entity. Accordingly, the reduced-order modeling technology [134,135] is available to reduce the computational effort for complex physical entities. Because physical entities operate in a complicated environment, finite element modeling, as the recommended physical modeling technology in this paper, demands expansion from a single structural field to a multi-physics coupled field and seamless integration with physical modeling tools. Other technologies for physical modeling including Denavit-Hartenberg (D-H) notation [25], object-oriented modeling method [71,73], data augmentation [136], and computational fluid dynamics (CFD) modeling [137,138] are shown in Fig. 11.

By integrating physical modeling technologies, the physical model is established and then analyzed in the physical modeling tools. Ansys [130,139] is a general-purpose finite element analysis software, which incorporates multiple physical fields. It delivers rich components for rapid finite element modeling and visualizes its analytical results through diverse charts. Also, as the mainstream finite element analysis software, Abaqus [106] is more adept at complex nonlinear problems and structural analysis. In addition, by creating custom physical modeling components, Simulink [36,44,90,122,131,140–142] can describe the physical concepts behind physical entities and the physical

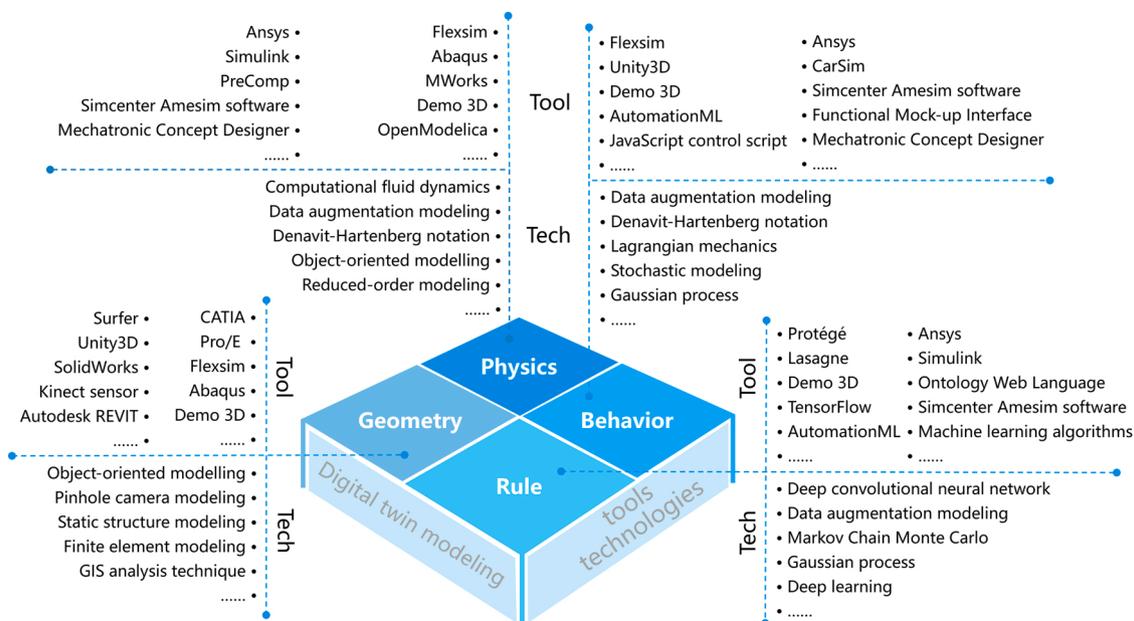


Fig. 11. Framework of technologies and tools in model construction.

connections between variables. Besides software, OpenModelica [23], an object-oriented and multi-domain modeling language, generally constructs physical models of mechanical, hydraulic, thermal, and electrical components. PreComp [131] is a code specializing in creating composite blades' structural properties like cross-coupled stiffness. The joint modeling of Simulink and Ansys is a promising approach. For the continuous physical modeling optimization, researchers can load the physical parameters Simulink generated onto finite element models of Ansys and feed the analytical results back into Simulink. Fig. 11 shows other physical modeling tools such as MWorks [71,73], Simcenter Amesim software [50], Flexsim [91], Demo 3D [125], and Mechatronic Concept Designer [127].

### 5.1.3. Enabling technologies and tools for behavioral modeling

Behavioral modeling technologies endow geometric and static physical models with dynamic functional behaviors. Based on the motion evolution in time and space studied by kinematic and dynamic analysis [103], each behavior of a physical entity can be described by differential equations [143]. For the behavioral model construction of the robot, Denavit-Hartenberg (D-H) notation [25] establishes the kinematic equations of the robot by formulating the end-effector position and attitude over the base coordinate system. In more complex scenarios, the dynamical equations for the physical entity's behaviors can be established based on Lagrangian mechanics [27]. Mechanical systems are commonly exposed to random disturbances that affect the motion evolutions of the system. As shown in Fig. 11, Stochastic modeling technology [144] depicts the impact of random factors through random variables and probability distributions. Hence, it can develop the stochastic behavioral models of the system under random disturbances. In summary, external random factors' impacts and internal geometrical or physical parameters should be considered simultaneously in future research on behavioral modeling technologies.

Behavioral modeling tools can actualize the behaviors of digital twin models in virtual environments. In web-based digital twin applications, JavaScript control script [103] based on ThreeJs can realize the functionality implementation, internal adjustment, external interaction, and event response of digital twin models according to the real scenario requirements. As object-oriented data modeling language, AutomationML [108,145] can be used to establish behavioral models of discrete object interoperability. Besides the programming languages above, Unity3D [89,126] is a real-time 3D platform for interactive content creation. It enables rapid behavioral modeling by mounting C#-based scripts to 3D objects in virtual scenes. Similarly, behavioral modeling can be implemented via other platforms shown in Fig. 11, such as Simcenter Amesim software [50], Ansys [68], Flexsim [91], CarSim [123], Demo 3D [125], and Mechatronic Concept Designer [127]. Unity3D is recommended for behavioral modeling due to its accessibility, versatility, and rich learning resources. However, the disadvantage is that the packages exported from Unity3D consume a larger portion of resources.

### 5.1.4. Enabling technologies and tools for rule modeling

Rule modeling technologies mine, extract and represent rules from historical data, operational logic, and expertise, grounding the digital twin model capabilities such as decision making, evaluation, and optimization. To realize the above capabilities, machine learning [146] is an effective technology that continuously improves and extends the performance of the digital twin model by reorganizing the existing knowledge structure. As shown in Fig. 11, Machine learning is formulated in various forms, each with its features. For instance, by combining Monte Carlo [38] and Markov chain [147], Markov Chain Monte Carlo (MCMC) overcomes the extremely complicated calculation of high-dimensional integration. MCMC can also realize the dynamic modeling where the sampling distribution varies as the simulation proceeds. With high modeling flexibility, Gaussian process [148] is available for regression, classification, feature extraction, and other such

machine learning tasks. Compared to artificial neural network (ANN) [143,149], deep convolutional neural network (DCNN) [148], can better simulate the brain while reducing overfitting. Noticeably, when using machine learning for rule modeling, the robustness of the rule model can be further improved by data augmentation [136]. As a subset of machine learning, deep learning is a recommended rule modeling technology with enhanced learning capability and broader coverage. However, the open and dynamic environments physical entities typically operate in impose challenges on deep learning technology.

Based on the logic, laws, and rules, rule modeling tools equip the digital twin model with intelligent capabilities and boost service quality. Ontology Web Language (OWL) [88,150,151] is used for a semantic description of ontologies. It draws on research in the AI field on knowledge representation, especially logical description, thus delivering a richer expressiveness for rule models of the digital twin model. TensorFlow [152] is a symbolic mathematical system based on data flow programming and extensively applied to implement various machine learning algorithms [153]. Its lightweight feature allows for faster creation and iteration of rule models. As a recommended rule modeling tool, Tensorflow is available in various fields of the digital twin research and can be deployed on various servers, PC terminals, and web pages. Fig. 11 shows other rule modeling tools like Ansys [68], Simcenter Amesim software [50], Demo 3D [125], Protégé [91,103,150,151], and Simulink [44,70,140,142].

### 5.2. Enabling technologies and tools in model assembly

After the individual models are constructed, the geometric connections between the models need to be established for more complex digital twin model. For model assembly, it is essential to consider the topological information between models based on the model hierarchy and assembly sequence. Static structure modeling technology [103] can address each unit-level digital twin model as an analysis class. Relationships in model assembly between classes can be established by describing the properties of each class and defining its state and operations. It is a new direction for model assembly research that enables rapid automatic assembly of 3D models based on the current 2D information such as facility plan drawings. Computer vision technology can extract information from images and process them for actual measurements. By combining it with optimization algorithms like particle swarm optimization algorithm [109], a rapid automatic assembly can be achieved. As shown in Fig. 12, Monte Carlo method [109] can also be used for model assembly.

In addition to technologies, some well-known commercial tools are available in the manufacturing field, such as Flexsim [91] and Demo3D [122] as shown in Fig. 12. During model assembly, these platforms can define the relative position of facilities to each other and the lateral area of aisles, thus maximizing space utilization. More generic model assembly tools are Unity3D [154] for PC and ThreeJS [103] for the web. Both can effectively address the geometric relations between digital twin models, including overlap, intersection, adjacency, etc. Simulink offers a number of toolboxes that can be used in different specialized areas. Simscape is the one specializing in multi-physics field modeling. With it, users can assemble components by physical connections between modular block diagrams. Based on the assembly topology of components, 3D model assembly can be done by importing 3D models. However, the tools presently accessible for model assembly are primarily for manufacturing. The development of model assembly tools for other fields like healthcare will broaden the potential applications of the digital twin technology.

### 5.3. Enabling technologies and tools in model fusion

After model assembly, technologies for model fusion specify the coupling relationships and modes between digital twin models. For example, semantic information modeling technology [13] establishes

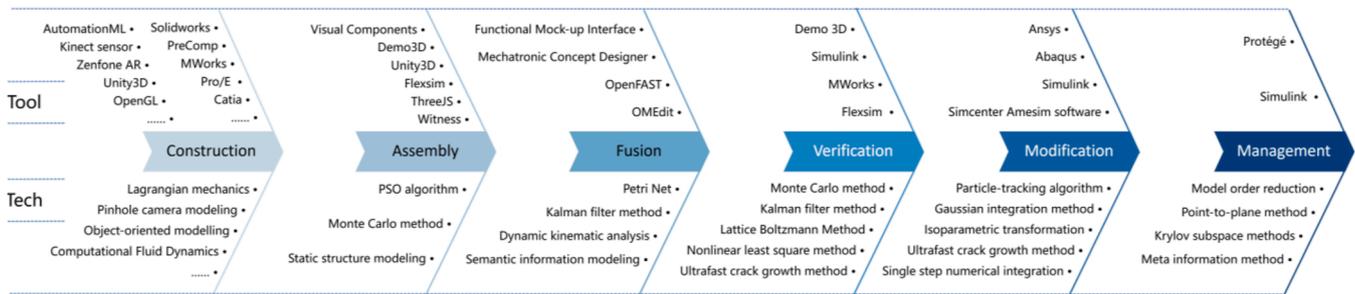


Fig. 12. Framework of technologies and tools in different digital twin modeling aspects.

the semantics and relationships of the data. The geometric information from model assembly and the process information from the production process documentation can be intelligently integrated to construct static coupling between digital twin models. Besides the static model fusion by semantic information modeling, Petri net technology [109] can represent the correlations between synchronous and concurrent logical behaviors to realize the dynamic fusion between digital twin models. The definition of the unidirectional and bidirectional coupling patterns between digital twin models in different domains [10] is also a technological concern for model fusion. In Fig. 12, Kalman filter method [45] is available for model fusion as well.

The fusion between digital twin models in different dimensions is also a concern for model fusion. As shown in Fig. 12, Functional Mock-up Interface (FMI) [155] defines a set of interfaces to enable model fusion in different dimensions. For example, FMI can connect the vehicle model created by CarSim and the functional model created by Simulink for joint simulation. In addition to model fusion in different dimensions, Mechatronic Concept Designer [127] seamlessly integrates disciplinary knowledge of mechanics, electricity, and automation based on unified modeling language from the beginning of machine design. In this way, functional models can be created and fused in different disciplines. As an integrated simulation tool of wind turbines, OpenFAST [131] can fuse aero, hydro, servo, and elastic models for simulation analysis. Based on OpenModelica, OMEdit [23] can provide interactive APIs which facilitate user-defined model extensions for model fusion. The current tools for model fusion in different dimensions and disciplines are limited to the engineering field, and tools for model fusion within and across other fields are scarce.

#### 5.4. Enabling technologies and tools in model verification

The purpose of model verification technologies is to verify whether and what degree the validity, accuracy, and applicability of the digital twin model running results address the physical production needs. It is worth noting that the choice of model verification technologies should be tailored to the actual application scenario. The small displacement torsor theory [109] and Monte Carlo method [109] can identify unsure model assembly positions. Hence, the assessment of the assembly accuracy can be performed by avoiding assembly errors. Static and dynamic equations analysis technology [106] can identify and analyze user-defined element errors, thus laying the groundwork for model modification. Lattice Boltzmann Method [156] can verify the permeability stability of porous media by analyzing models at the pixel level. However, the existing model verification technologies for the digital twin model lack mature technological systems and generic theoretical guidance, which are advancement directions for future research. As shown in Fig. 12, other model verification technologies in the literature reviewed include nonlinear least square method [23], Kalman filter method [45], and Ultrafast crack growth methodology [157].

In model verification, the running results of the assembled or fused digital twin models are required to be compared with the actual values. For this reason, some software is available, as shown in Fig. 12,

including MWorks [71,73], Simulink [23,25,77,140], Flexsim [91], and Demo 3D [125]. MWorks is a platform for developing the multi-field engineering system. It can verify the applicability of parameter values to real scenarios by a value comparison between the model variables in simulation results and the measured metric variables. Simulink can examine simulation results and troubleshoot abnormal model behavior via its graphical debugger and profiler. Meanwhile, Simulink's model analysis and diagnostic tools can verify the model's consistency and pinpoint errors in the model. Notably, Simulink is more suitable for beginners to start with its graphical programming feature.

#### 5.5. Enabling technologies and tools in model modification

After model verification, the digital twin model requires modification with adequate technologies in a targeted manner so that the digital twin model meets the application requirements. For instance, since the element property of the digital twin model was expressed by the numerical matrix, isoparametric transformation [106] and the Gaussian integration method [106] were used to adjust each element of the numerical matrix to modify the model properties. Ultrafast crack growth methodology [157] computed the equivalent constant amplitude spectra to rapidly correct a mountain of crack growth realities on a single processor. Other technologies for model modification are shown in Fig. 12, including single step numerical integration method [158] and particle-tracking algorithm [159]. However, the universal technological framework for model modification is lacking, causing increased trial-and-error costs and even failure to derive the expected modification results.

In Fig. 12, tools including Abaqus [25], Simcenter Amesim software [50], Ansys [77,160], are viable for model modification. Besides the powerful finite element modeling capabilities, Abaqus and Ansys can prioritize the model variables with higher sensitivity by calculating their sensitivity towards the objective functions and state variables. Then, the modification iterations are carried out with appropriate modification technologies to obtain a more accurate finite element model. As an electromechanical system simulation platform, Simcenter Amesim software enables performance verification and control modification for the digital twin model in an iterative process. This iterative process can also be performed jointly with Simulink through interfaces. Individual model modification tool frequently targets only one model dimension or one field. For multidimensional digital twin models in multiple fields, collaborative modification across multiple tools is desired.

#### 5.6. Enabling technologies and tools in model management

As the digital twin modeling progress, the model data accumulated and the modeling knowledge base gradually formed necessitate management for later reference and reuse. However, multidimensional and multilevel digital twin models with multidisciplinary knowledge are unfavorable for storage, exchange, and reuse. The model order reduction technology [107] simplifies the complexity of the digital twin model and conserves its essential features. As a result, it facilitates model

storage and exchange while componentizing the model for easy reuse in other digital twin modeling aspects. Because requirements for the model vary with each modeling aspect, it may require model reuse and fine-tuning based on existing model data in the model library. Krylov subspace method [107] can discretize the model state equations into several small sets of equations that simply need targeted modifications to certain sub-equations for the model reuse and fine-tuning. The Point-to-plane method [109] develops mapping relationships between simpler point cloud models and tanglesome geometric elements of the digital twin model. Therefore, the simplification of digital twin model management is achievable in the geometric dimension. The model management technology should not be restricted to models and modeling, but also extend to the services they provide. As shown in Fig. 12, the meta-information driven method [114] can standardize model exchanging services by (de-) serializing dynamic model information.

In model management, the service towards digital twin model for users derived from model management can be obtained by tools. In Fig. 12, Protégé [24,29] is available for model exchange which can import and store models in various formats such as XML, UML, and RDF. Besides, users can utilize OWL 2 accelerated by Protégé [24] for rapid virtualization and parameter configuration. Remarkably, by representing models as module diagrams, Simulink can visually manage them through an interactive graphical editor. Simscape of Simulink can divide the model based on its hierarchical nature into equipment level, topology level, and multi-physics field level for efficient management. Meanwhile, Simscape provides rapid modification service of part parameters to respond to changing mechanical modeling needs. However, the current model management tools are not well connected with tools in the other digital twin modeling aspects. The disconnection prevents from reusing the model data and modeling knowledge the digital twin modeling generated and hinders the extension of services digital twin models provided.

### 5.7. Conclusions for technologies and tools above

The digital twin modeling theoretical system has been proposed and investigated from the six modeling aspects. However, the technological standards and systems for each modeling aspect and the entire modeling process are still waiting to be elaborated to address the technological issues involved in the actual modeling procedures.

At each modeling aspect within the digital twin modeling theoretical system, there are related software tools to perform the required works. Nevertheless, nowadays, no tool integrates the functionalities of the previously described modeling software in each aspect to accomplish the all-process digital twin modeling. Therefore, in future research, the development of an all-process digital twin modeling tool needs to move forward under the guidance of the digital twin modeling theoretical system.

## 6. Observations and recommendations

This work provides a thorough review of over 300 publications. Therein, Fastidious data statistics and analytics on various properties of the digital twin model and dissimilar modeling aspects of the digital twin modeling have been conducted. Accordingly, there are some observations and recommendations expounded as follows:

### 6.1. Hierarchy of the digital twin model

In terms of the digital twin shop floor in manufacturing, diverse production units can be hierarchized depending on their production characteristics and functionalities. A reasonable hierarchical structure of digital twin model contributes to the clarified and efficient organization, coordination, and management. In other application fields besides manufacturing, the digital twin technology has already blossomed.

However, no literature researches the hierarchy of the digital twin model. Consequently, the correlations between objects in the same scenario, let alone different scenarios and even application fields, are not explicit. Since the hierarchy is underlying the myriad correlations between digital twin models, it is crucial to establish the hierarchical structure for the digital twin model. In future research, the macroscopic hierarchical system for digital twin models in various application fields and the microcosmic hierarchical structure for model dimensions within an individual one are required to be established.

### 6.2. Discipline of the digital twin model

From traditional to cutting-edge disciplines, from engineering to medical disciplines, the digital twin has embraced a vast disciplinary spectrum. Nevertheless, the disciplines involved in digital twin models are isolated from each other in current research. The conversion of disciplinary knowledge to accurate and efficient digital twin modeling is insufficient. Thus, future works entail gap-filling between different disciplines for deeper interdisciplinary integration. The seamless disciplinary knowledge transformation to satisfy the practical modeling needs is also a future research direction for the digital twin technology.

### 6.3. Dimension of the digital twin model

A comprehensive study of the digital twin model from the perspective of the four model dimensions of the virtual model, i.e., geometry, physics, behavior, and rule. By portraying physical entities in these four dimensions, the digital twin model can be better characterized and the corresponding research can be more targeted. But for other areas, the analysis of the attributes and functions of the virtual model based on the four model dimensions is absent. In some cases, the existing portrayal of the digital twin model is relatively one-sided and cannot incorporate and reflect all dimensions of its virtual model, which prevents the digital twin model from realistically mapping the physical entity. At the same time, studies that address the fusion of these four model dimensions are not available in the literature. Future research for digital twin models should be grounded on these four dimensions, to accurately mirror the physical entities and fully utilize the digital twin model effectiveness. The fusion of different model dimensions requires consideration as well.

### 6.4. Universality of the digital twin model

By classifying the digital twin models into specific and generic ones according to their characteristics and scope of application, we can enable digital twin models to deliver more dedicated and distinct services for production and life. So far, no article has elaborated on the criteria for categorizing digital twin models based on their universality. For digital twin models with different universality, the dedicated and all-purpose modeling methodologies, modeling theoretical systems, and modeling tools remain unavailable. At present, only the five-dimensional digital twin model is a general one applicable to all application fields. Future research requires distinguishing the distinct nature of digital twin models with different universality. Accordingly, more effective specific and generic digital twin models will be created by employing appropriate modeling methodologies and tools.

### 6.5. Functionality of the digital twin model

Digital twin models can provide a wide range of effective functions based on certain production requirements. However, the current literature suffers from insufficient consideration of the multidimensional Spatio-temporal scales and extrinsic factors for physical objects. Consequently, the inconsiderateness risks undermining the robustness of the functionality of the digital twin model during the real-time dynamic operation and during the evolutionary process advancing over time. To take full advantage of the digital twin utility, future research on digital

twin model functions is expected to thoroughly consider the practicalities of the corresponding physical objects and operational scenarios.

### 6.6. Modeling aspects of the digital twin modeling

The six modeling aspects within the digital twin modeling theoretical system present an essential reference for whole-process accurate digital twin modeling. Unfortunately, existing digital twin modeling studies have incomplete and unbalanced coverage of these six modeling aspects. The digital twin modeling in some fields is merely confined to model construction. Research on the modeling aspects besides model construction still stagnated at a relatively rudimentary level without well-developed systematic methodologies. Moreover, no literature empowers all the six modeling aspects with concrete application cases. In future work, we need to strengthen the study of the six modeling aspects intensively and extensively. It is paramount to garner attention to the consistency, coherence, and collaboration between the modeling aspects in their implementation. In this way, the connotation and applicability of the six modeling aspects can be continuously refined and expanded in future practices.

### 6.7. Technologies and tools for the digital twin modeling

A wide variety of modeling technologies and tools are accessible today for various modeling aspects. Nevertheless, a majority of present modeling technologies and tools specialize in model construction, leaving insufficient options for subsequent aspects. There is a dearth of a technology and tool system or criterion to instruct the employment of the technologies and tools used in each modeling aspect. The correlation between technologies and tools utilized in the same modeling aspect is ambiguous. Furthermore, the technologies or tools deployed in different modeling aspects are separated from each other and without coherence between them. The separation and incoherence will lead to unnecessary costs in time and money and increase modeling errors. It is urgent to develop a modeling technology system and an integrated software toolkit or platform, which incorporate all the six modeling aspects. As a result, researchers can perform the digital twin modeling at minimal cost and enable maximum access to the corresponding service provided by high-fidelity digital twin models.

## 7. Conclusions and future work

Since the first introduction of the terminology and the first application in aerospace, the digital twin has been rapidly advancing from concept to application in recent years. Along with this, the research and applications of digital twins are increasingly in-depth and specific. Therefore, the digital twin model as the cornerstone of the digital twin technology and the digital twin modeling as the golden touch for this cornerstone have received more and more attention. This paper provides a systematic review of over 300 previous publications related to the digital twin modeling and its outcome, digital twin models. The primary contributions of this review can be concluded as follows:

- 1) It delivers holistic and fine-grained statistics and analytics on the state-of-the-art research of the digital twin model in terms of the application field, hierarchy, discipline, dimension, universality, and functionality.
- 2) It reviews and analyzes the current research on the digital twin modeling at different modeling aspects. In summary, present digital twin modeling studies are mainly staying on model construction and modeling integrity is a pressing issue.
- 3) It summarizes the technologies and tools used in different modeling dimensions and modeling aspects and provides practical use cases as well as general directions for enabling ones, which are feasible and promising in corresponding digital twin modeling aspects.

Even though research for the digital twin is surging, current works on the digital twin modeling are still in the infancy of rapid development. Many imperatives should be resolved to refine the capability of the digital twin modeling in portraying the digital twin models and improving their feasibility in practice. For instance, an integrated software platform incorporating all modeling aspects is in urgent demand. Accordingly, this paper serves as a reference for relevant researchers to explore future orientations of the digital twin modeling.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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